

# Near-Threshold Production of $\eta$ -Mesons

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## Abstract

It is shown that the striking energy variation in the  $p d \rightarrow {}^3\text{He} \eta$  cross section near threshold is probably due to a final state interaction associated with a large (complex)  $\eta - {}^3\text{He}$  scattering length. The consequences of this hypothesis are studied for the production of the meson in the  $\eta - {}^4\text{He}$  and  $\eta - {}^7\text{Be}$  channels.

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The cross sections for the production of  $\eta$  and  $\pi^0$  mesons *via* the  $p d \rightarrow {}^3\text{He} \eta$  and  $p d \rightarrow {}^3\text{He} \pi^0$  reactions behave very differently close to their respective thresholds. Taking out the kinematic factor of the ratio of the outgoing to the incident centre-of-mass momenta, we can define an average squared amplitude through

$$|f_{\eta(\pi)}|^2 = \frac{p_p}{p_{\eta(\pi)}} \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}}. \quad (1)$$

For  $\eta$ -production,  $|f_\eta|^2$  decreases by over a factor of three between threshold and an  $\eta$  centre-of-mass momentum of  $p_\eta = 70$  MeV/c, while remaining essentially independent of production angle [1, 2]. This corresponds to a change in beam energy of less than 10 MeV. In contrast,  $|f_\pi|^2$  shows a strong angular variation, with the ratio of the forward to backward cross section already attaining a factor of three by  $p_\pi = 20$  MeV/c. However the angular average has a much weaker energy dependence than for the  $\eta$  case [3, 4].

This striking difference in the angular distribution is almost certainly due to the basic meson-nucleon interaction. The  $\pi N$  interaction is governed by a P-wave resonance, the  $\Delta(1232)$ , with only a very weak S-wave. The strong angular dependence seen in the  $p d \rightarrow {}^3\text{He} \pi^0$  cross section is then a result of an interference of the large P-wave with an S-wave which is only significant within a few MeV of threshold [5]. On the other hand the most prominent feature of the low energy  $\eta N$  interaction is an S-wave resonance, the  $N^*(1535)$ , so that the low energy  $\pi^- p \rightarrow \eta n$  reaction shows only a comparatively weak angular dependence [6]. It is our contention that this strong S-wave interaction is also responsible for the rapid energy variation of the near-threshold  $p d \rightarrow {}^3\text{He} \eta$  cross section.

Whereas the  $p d \rightarrow {}^3\text{He} \pi^0$  cross section and deuteron tensor analysing power are both well described at low energies by a model involving a spectator nucleon [5], such an approach fails for the  $p d \rightarrow {}^3\text{He} \eta$  reaction and three-nucleon mechanisms have been suggested as the origin [7, 8]. Though estimates of the effects of such terms within a semi-phenomenological model do reproduce some of the features of the data, they predict little energy dependence of the amplitude within a few MeV of threshold [9, 10]. It should though be noted that these models are perturbative, treating all interactions only to lowest order, and this might not be justified for low energy  $\eta$ -nucleon scattering where the S-wave is very strong.

A common approximation [11], in the case of a weak transition to a

channel with a strong final-state interaction (FSI), yields an S-wave threshold enhancement factor of the amplitude  $f$ ,

$$f = \frac{f_B}{p a \cot\delta - ip a} , \quad (2)$$

where the amplitude  $f_B$  is slowly varying near threshold, and  $\delta$  and  $a$  are the S-wave phase shift and scattering length in the exit channel, for which the centre-of-mass momentum is  $p$ . At low energies it is often sufficient to take

$$f \approx \frac{f_B}{1 - ip a} . \quad (3)$$

The approximation leading to this expression corresponds to imposing unitarity with constant K-matrix elements, *i.e.* neglecting effective range effects [12]. In view of our dearth of knowledge about the low energy  $\eta$ -nucleon (nucleus) interaction, it is pointless trying to go further at present. It should be noted that the effects of the  $S_{11}$  resonance are felt in the final state interaction factor rather than in the  $f_B$  term.

Bhalerao and Liu [13] analysed the  $\pi N$  and  $\eta N$  coupled channels around the  $\eta$  threshold within an isobar model and extracted a value for the  $\eta$ -nucleon scattering length of  $a(\eta N) = (0.27 + i0.22)$  fm. However a value which is more consistent with our later use of it may be obtained by applying eq. (3) directly to  $\pi^- p \rightarrow \eta n$  data.

Using detailed balance and the optical theorem, a lower bound on the imaginary part of  $a(\eta N)$  is provided by the threshold  $\pi^- p \rightarrow \eta n$  cross section:

$$\text{Im}[a(\eta N)] \geq \frac{3}{8\pi} \frac{p_\pi^2}{p_\eta} \sigma_{\text{tot}}(\pi^- p \rightarrow \eta n) . \quad (4)$$

The data of reference [6] require  $\text{Im}[a(\eta N)] \geq (0.28 \pm 0.04)$  fm which, though a little larger than the value extracted in [13], is compatible with it. Other channels, such as  $N\pi\pi$ , are also open at the  $\eta$  threshold and these must add to the inelasticity [14]. We therefore take  $\text{Im}[a(\eta N)] = 0.30$  fm, though this might be an underestimate.

Even after neglecting effective range effects, the  $\pi^- p \rightarrow \eta n$  energy dependence is not sufficient to determine both the real and imaginary parts of the scattering length but once the imaginary part is fixed from the transition strength, the fit shown in fig. 1 leads to

$$a(\eta N) = (0.55 \pm 0.20 + i0.30) \text{ fm} . \quad (5)$$

The sign of the real part is ambiguous and we have chosen it to be attractive to be consistent with the work of Bhalerao and Liu [13]. It should be noted that, though the magnitude is twice the value they found, any P-wave contributions to the cross section would have the effect of reducing the slope in energy and hence giving a too low an estimate for  $\text{Re}[a(\eta N)]$ .

In impulse approximation the  $\eta^3\text{He}$  scattering length is essentially three times that of  $\eta N$  but there are large corrections to this simple ansatz due to the strength of the interaction. A more reliable starting point is to consider the lowest order  $\eta^3\text{He}$  optical potential for which

$$2m_{\eta N}^R V_{\text{opt}}(r) = -4\pi A \rho(r) a(\eta N) , \quad (6)$$

where  $m_{\eta N}^R$  is the  $\eta$ -nucleon reduced mass and  $A$  ( $= 3$ ) the mass number. Resolving the variable phase equation [15] for this potential, using a Gaussian nuclear density corresponding to an rms radius of 1.9 fm, leads to a scattering length of

$$a(\eta^3\text{He}) = (-2.31 + i2.57) \text{ fm} . \quad (7)$$

The sign of the real part indicates the possible presence of a ‘bound’  $\eta$  state for a much lighter nucleus than that found by Haider and Liu [16], but the large imaginary component in the scattering length limits its possible significance.

The prediction of the shape of the energy dependence of the  $p d \rightarrow {}^3\text{He} \eta$  cross section using the simplified FSI formula of eq. (3) is shown as the dashed line in fig. 2, where it is compared with the pioneering SPES4 data [1] and the preliminary SPES2 values [2] which were used to fix the overall normalisation parameter. The lowest SPES2 point was taken at an average energy only 200 keV above threshold and is subject to large systematic uncertainties due to the width of the beam. The energy loss in the target alone was up to 270 keV [2] and this influences both the value of  $d\sigma/d\Omega$  and  $p_\eta$  in eq. (1). If we exclude this doubtful point then the agreement with experiment, which is stable to modest changes in  $\text{Im}[a(\eta N)]$ , is impressive. Though numerically a little fortuitous, in view of the corrections which might be important to the lowest order optical potential, nevertheless it indicates that the rapid fall of the amplitude with energy might indeed be associated with a strong  $\eta^3\text{He}$  final state interaction.

Once we have the potential of eq. (6) then we can calculate the phase shift at all energies, which enables us to use the more general formula of

eq. (2) rather than the constant scattering length version of eq. (3). This in fact makes very little difference, as can be seen from the solid line in fig. 2.

It is not possible to extract directly values of the real ( $a_R$ ) and imaginary ( $a_I$ ) parts of the  $\eta$ - $^3\text{He}$  scattering lengths independently from the present data using eq. (3). Taking only the highest 6 SPES2 points [2], a  $\chi^2$  minimisation shows that these parameters are roughly correlated in the form

$$a_R^2 + 0.449a_I^2 + 4.509a_I = 21.44. \quad (8)$$

This at least demonstrates that either the real or imaginary part of the scattering length has to be very large.

A large scattering length, associated with a ‘bound’  $\eta$ - $^3\text{He}$  system, also seems to be required [17] to explain the cusp-like structure seen for near-threshold production of states  $X$  in the  $p d \rightarrow ^3\text{He} X$  reaction for masses close to that of the  $\eta$ -meson [18].

In their microscopic model, Laget and Lecolley [7] include only a small amount of  $\eta$ -nucleon rescattering and as a consequence underestimate severely the energy dependence near threshold. Taking both S and D states in the nuclear wave function, their graphs are consistent with an  $\eta$ - $^3\text{He}$  scattering length of modulus 1.6 fm. This leads to only a 25% decrease in  $|f|^2$  by  $p_\eta = 0.35 \text{ fm}^{-1}$  as compared to the factor of three shown in fig. 2.

The S-wave FSI enhancement factor of eq. (2) is independent of the entrance channel though the particular nuclear reaction would influence the amount of P and higher waves present. It should therefore be applicable also to the  $\pi^-$ - $^3\text{He} \rightarrow \eta$ - $^3\text{H}$  reaction. Unfortunately the lowest energy for which this has been measured [19] corresponds to  $p_\eta = 0.41 \text{ fm}^{-1}$ , which is just off the scale of fig. 2 ! This may be why Liu [20] did not note any significant FSI distortion, though it must be stressed that his effective  $\eta$ -nucleus potential was also rather weaker. Our analysis indicates that it could be very interesting to continue the experiment closer to threshold.

The success of our simple interpretation encourages us to look for other nuclear reactions in which  $\eta$ -mesons are produced coherently. Data exist in the case of  $d d \rightarrow ^4\text{He} \eta$ , but only away from the threshold region [21]. Taking an rms radius of 1.63 fm, the  $\eta$ - $^4\text{He}$  potential is stronger but of shorter range than that for  $\eta$ - $^3\text{He}$  and this ‘binds’ further the highly inelastic  $\eta$ - $^4\text{He}$  state. The predicted scattering length of  $(-2.00 + i0.97) \text{ fm}$  corresponds to a somewhat less steep energy dependence than that found for  $\eta$ - $^3\text{He}$  production.

Including also effective range effects through eq. (2), the decrease in  $|f|^2$  between  $p_\eta = 0.1$  and  $0.4 \text{ fm}^{-1}$  is expected to be about 2.8 for  ${}^3\text{He}$  but only 1.9 for  ${}^4\text{He}$ .

The only other case where coherent  $\eta$  production on nuclei has been studied is that of  $p \, {}^6\text{Li} \rightarrow \eta \, {}^7\text{Be}^*$  [22], though the energy resolution obtained by detecting the  $\eta$  through its  $2\gamma$  decay mode was insufficient to isolate individual states in the final  ${}^7\text{Be}$  nucleus. Since the optical potential of eq. (6) predicts a scattering length of  $(-2.92 + i1.21) \text{ fm}$  and the typical  $\eta$  centre-of-mass momentum in this experiment was  $p_\eta \sim 0.5 \text{ fm}^{-1}$ , these data lie outside the FSI peak. It might be advantageous to study the cross sections at say 1–2 MeV above the threshold for the excitation of a particular level.

In summary, the very simplified analysis presented here shows that the strong energy dependence associated with coherent  $\eta$  production on nuclei is consistent with a large (complex)  $\eta$ -nucleus scattering length. Theoretical models of near-threshold production which ignore FSI effects must therefore be treated with caution.

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Figure 1: The square of the  $\pi^- p \rightarrow \eta n$  amplitude defined by eq. (1), and extracted from the total cross section data of ref. [6], as a function of the  $\eta$  centre-of-mass momentum  $p_\eta$ . The solid line is a fit using eq. (3) with the imaginary part of the  $\eta N$  scattering length constrained by unitarity. This leads to the parameters of eq. (5). The dashed line is the best fit with  $\text{Im}[a(\eta N)] = 0$ .

Figure 2: The square of the  $p d \rightarrow {}^3\text{He} \eta$  amplitude defined by eq. (1) and extracted from the cross section data of SPES2 [2] (circles) and SPES4 [1] (crosses) as a function of the  $\eta$  centre-of-mass momentum  $p_\eta$ . The lowest SPES2 point is subject to large systematic errors due to beam width effects, including energy losses in the target. The dashed curve is the prediction of eq. (3) with the scattering length of eq. (7) derived from an optical potential. The solid curve is based on eq. (2) and includes effective range effects. In both cases the overall normalisation is a free parameter.